INDIAN SCHOOL MUSCAT
FIRST PRE-BOARD EXAMINATION
MATHEMATICS

CLASS: XII   Sub. Code: 041   Time Allotted: 3 Hrs.

07.03.2021   Max. Marks: 80

General Instructions:

1. This question paper contains two parts A and B. Each part is compulsory. Part A carries 24 marks and part B carries 56 marks.
2. Part-A has Objective Type Questions and Part-B has Descriptive Type Questions.
3. Both Part A and Part B have internal choices.

PART – A:

1. It consists of two sections- I and II.
2. Section I comprises of 16 questions of 1 mark each.
3. Section II contains 2 case study questions. Each case study comprises of 5 case-based MCQs. An examinee is to attempt any 4 out of the 5 MCQs.

PART – B:

1. It consists of three sections – III, IV and V.
2. Section III comprises of 10 questions of 2 marks each.
3. Section IV comprises of 7 questions of 3 marks each.
4. Section V comprises of 3 questions of 5 marks each.
5. Internal choice is provided in 3 questions of Section III, 2 questions of Section IV and 3 questions of Section V. You have to attempt only one of the alternatives in all such questions.

<u>Note : One graph sheet to be provided.</u>

PART A
SECTION - I

Questions 1 to 16 carry 1 mark each.

1. Find the principal value of : $\cos^{-1}\left|\cos\left(-\frac{7\pi}{3}\right)\right|$ (1)

   (OR)

   Express in simplest form : $\cot^{-1}\left(\frac{\cos x}{1-\sin x}\right)$, $\quad x < \frac{\pi}{2}$

2. For any 2×2 matrix A, if $A(adj\ A) = \begin{vmatrix} 10 & 0 \\ 0 & 10 \end{vmatrix}$, then find $|A|$. (1)

3. If $\left|\vec{a} \times \vec{b}\right|^2 + \left|\vec{a}.\vec{b}\right|^2 = 144$ and $|\vec{a}| = 4$, find the value of $|\vec{b}|$. (1)

4. What is the equation of the normal to the curve $y = sinx$ at $(0, 0)$? (1)

5. If $y = e^{sin\sqrt{x}}$, find $\frac{dy}{dx}$. (1)

6. If $\begin{vmatrix} 0 & a & 2 \\ 3 & b & -5 \\ c & 5 & 0 \end{vmatrix}$ is a skew-symmetric matrix, find the values of a, b and c. (1)

(OR)

For the matrix $A = \begin{vmatrix} 3 & 1 \\ 7 & 5 \end{vmatrix}$, find x and y so that $A^2 + xI = yA$. (1)

7. If $y = log\sqrt{\left(\frac{1-x^2}{1+x^2}\right)}$, find $\frac{dy}{dx}$. (1)

8. If $A = \begin{vmatrix} 2 & 3 \\ 1 & 0 \end{vmatrix} = P + Q$, where P is symmetric and Q is skew-symmetric matrix, then find the matrix Q. (1)

9. Find the distance of the plane $\vec{r}.\left(2\hat{\imath} + 3\hat{\jmath} - 6\hat{k}\right) = 7$ from the origin. (1)

10. If A_{ij} is the cofactor of the element a_{ij} of the determinant $\Delta = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$, find the value of $a_{32} \times A_{32}$. (1)

11. Evaluate : $\int cos^3 x . e^{log\,sinx}\, dx$ (1)

12. What is the value of the constant of integration in the particular solution of the differential equation $\frac{dy}{dx} = \frac{2x}{y^2}$, if $f(-2) = 3$ (1)

13. Find the value of : $\hat{\imath}.\left(\hat{\jmath} \times \hat{k}\right) + \hat{\jmath}.\left(\hat{k} \times \hat{\imath}\right) + \hat{k}.\left(\hat{\jmath} \times \hat{\imath}\right)$ (1)

14. Find the area of the region bounded by the curve $y = |x - 2|$, the lines $x = 1$, $x = 2$ and the x-axis. (1)

(OR)

Find the area bounded by the line $y - 1 = x$, the x-axis and the lines $x = -1$ and $x = 3$.

15. Evaluate : $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} sin^5 x\, dx$ (1)

16. Two dice are thrown. Find the probability that the numbers appeared have sum 8, if it is known that the second die always exhibits 4. (1)

Case study-based questions are compulsory. Attempt any four sub parts of each question. Each sub part carries 1 mark.

7. The Transamerica Pyramid, shown below, is an office building in San Francisco. It stands 853 feet tall and is 145 feet wide at its base. Imagine that a coordinate plane is placed over a side of the building. The graph on the left in the image below represents a cross section of the tower (graph is not proportional to actual height and width) and is defined by the function,

$f(x) = -|x - 50| - |x - 100| + 150,$ where x axis represents the width and y axis represents the height.
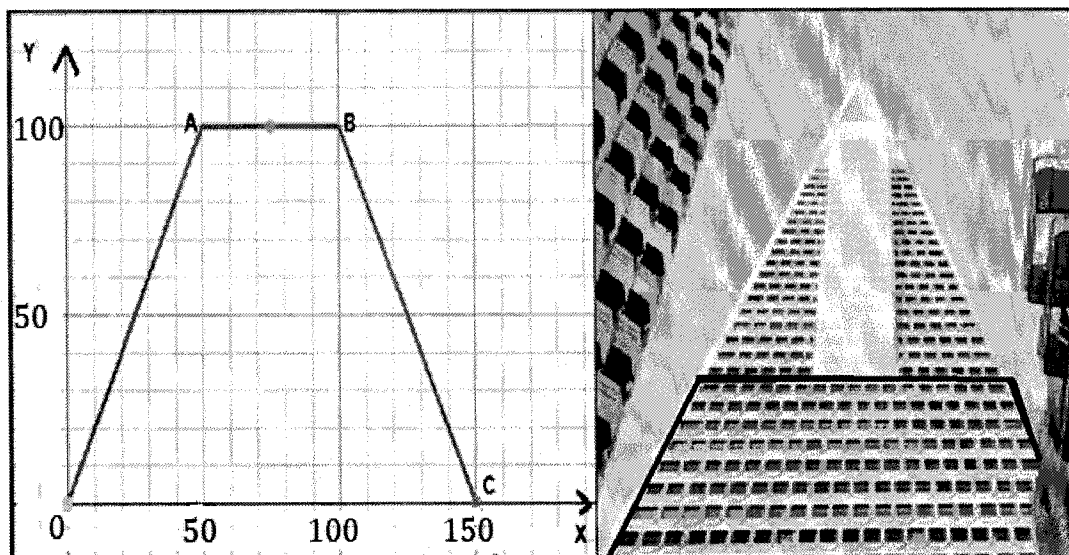
Based on the above information, answer any four of the following questions:

(i) Which among the following is the correct statement?

(A) $f(x)$ is discontinuous at $x = 50$ and $x = 100$. (1)

(B) $f(x)$ is continuous at only $x = 50$ and $x = 100$.

(C) $f(x)$ is continuous at all points in its domain.

(D) $f(x)$ is discontinuous at all points in its domain.

(ii) Find $f'(x)$ when $x \in (50, 100)$

(A) 5   (B) 0   (C) -1   (D) 10 (1)

(iii) Which statement among the following is correct?

(A) $f(x)$ is differentiable everywhere.   (B) $f(x)$ is not differentiable in $(0, 50)$ (1)

(C) $f(x)$ is not differentiable in $(50, 100)$ (D) $f(x)$ is not differentiable at $x = 50$ and $x = 100$

(iv) Find $\dfrac{dy}{dx}$ when $x = 120$

(A) -2 (B) 2   (C) 0   (D) 15 (1)

(v) Find $\dfrac{d^2y}{dx^2}$ when $x = 40$

(A) -2   (B) 2   (C) 0   (D) 15 (1)

18. A boy observes a pedestrian bridge over the roadway and realizes that the overpass is an excellent example of skew lines. He considers the roadway as the line L₁ and position of a car with coordinates A(1, 2, 1) on it. Similarly, he considers the walkway as the line L₂ and position of a man with coordinates B(2, -1, -1) on it. Also, $\vec{b}_1 = \hat{\imath} - \hat{\jmath} + \hat{k}$ and $\vec{b}_2 = 2\hat{\imath} + \hat{\jmath} + 2\hat{k}$ are vectors along L₁ and L₂ respectively.
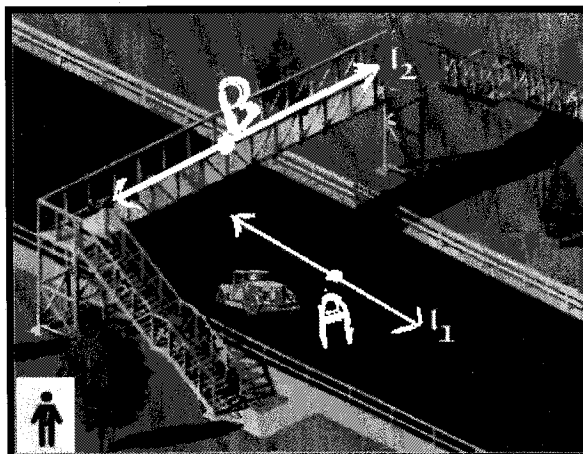
Based on the above information, answer the following :

(i) Vector equation of the line L₁ is : (1)

(A) $\vec{r} = (\hat{\imath} - \hat{\jmath} + \hat{k}) + \lambda(\hat{\imath} + 2\hat{\jmath} + \hat{k})$ (B) $\vec{r} = (\hat{\imath} - \hat{\jmath} + \hat{k}) + \lambda(2\hat{\imath} - \hat{\jmath} - \hat{k})$

(C) $\vec{r} = (\hat{\imath} + 2\hat{\jmath} + \hat{k}) + \lambda(\hat{\imath} - \hat{\jmath} + \hat{k})$ (D) $\vec{r} = (2\hat{\imath} - \hat{\jmath} + \hat{k}) + \lambda(2\hat{\imath} + \hat{\jmath} + 2\hat{k})$

(ii) The vector perpendicular to each of the vectors $\vec{b}_1$ and $\vec{b}_2$ is (1)

(A) $-3\hat{\imath} + \hat{\jmath}$ (B) $\hat{\imath} - 3\hat{\jmath}$ (C) $\hat{\imath} + 3\hat{\jmath}$ (D) $-3\hat{\imath} + 3\hat{k}$

(iii) Cartesian equation of the line L₂ is : (1)

(A) $\dfrac{2-x}{2} = \dfrac{y+1}{1} = \dfrac{z+1}{2}$ (B) $\dfrac{x-1}{1} = \dfrac{y-2}{-1} = \dfrac{z-1}{1}$

(C) $\dfrac{x-1}{2} = \dfrac{y-2}{1} = \dfrac{z-1}{2}$ (D) $\dfrac{x-2}{2} = \dfrac{y+1}{1} = \dfrac{z+1}{2}$

(iv) Find the direction cosines of the line L₁. (1)

(A) $\dfrac{1}{\sqrt{3}}, \dfrac{1}{\sqrt{3}}, \dfrac{1}{\sqrt{3}}$ (B) $\dfrac{1}{3}, \dfrac{1}{3}, \dfrac{1}{3}$ (C) $\dfrac{1}{\sqrt{3}}, -\dfrac{1}{\sqrt{3}}, \dfrac{1}{\sqrt{3}}$ (D) $\dfrac{1}{3}, -\dfrac{1}{3}, \dfrac{1}{3}$

(v) The shortest distance between the given two skew lines is (1)

(A) $3\sqrt{2}$ (B) $\dfrac{3\sqrt{2}}{2}$ (C) $\sqrt{2}$ (D) $\sqrt{3}$

PART B
SECTION – III
Questions 19 to 28 carry 2 marks each.

19. Show that the relation on ℝ defined as $R = \{(a, b): a \le b^3\}$ is not transitive. (2)

20. Simplify : $cos\left[2\cot^{-1}\sqrt{\dfrac{1-x}{1+x}}\right] + x$ (2)

21. If the line drawn from the point (-2, -1, -3) meets a plane at right angle at the point (1, -3, 3), find the equation of the plane. (2)

22. Find a vector of magnitude 9, perpendicular to each of the vectors $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$, where $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$. (2)

(OR)

If the sum of two unit vectors is a unit vector, prove that the magnitude of their difference is $\sqrt{3}$.

23. Verify that the function $xy = \log y + C$ is a solution of the differential equation $y' = \dfrac{y^2}{1-xy}$ (2)

24. Evaluate : $\int \dfrac{(x+6)}{(x+9)^4} e^x \, dx$ **(OR)**   Evaluate : $\int_0^{\frac{\pi}{2}} \dfrac{2^{sinx}}{2^{sinx} + 2^{cosx}} \, dx$ (2)

25. A bag contains 3 oranges and 4 apples. Two fruits are randomly drawn. (2)
Let X represent the number of oranges drawn. Find the probability distribution of X.

26. Find equation of normal to the curve $y = x + \dfrac{1}{x}$, $x > 0$ which is perpendicular to the line $3x - 4y = 7$. (2)

27. If $x = ae^t(sint + cost)$ and $y = ae^t(sint - cost)$, prove that $\dfrac{dy}{dx} = \left(\dfrac{x+y}{x-y}\right)$ (2)

(OR)

If $y = \tan^{-1}\left(\dfrac{a}{x}\right) + \log\sqrt{\dfrac{x-a}{x+a}}$, prove that $\dfrac{dy}{dx} = \dfrac{2a^3}{x^4 - a^4}$

28. Find the absolute maximum and the absolute minimum values of the function: (2)

$$f(x) = 12x^{\frac{4}{3}} - 6x^{\frac{1}{3}} \quad \text{on } [-1, 1]$$

SECTION – IV

Questions 29 to 35 carry 3 marks each.

29. Let $f: [0, \infty) \rightarrow [0, \infty)$ be given by $(x) = \dfrac{2x}{1+2x}$. Is f a bijective function? Justify your answer. (3)

30. If $A = \begin{vmatrix} 2 & 3 \\ 1 & -4 \end{vmatrix}$, $B = \begin{vmatrix} 1 & -2 \\ -1 & 3 \end{vmatrix}$; verify whether or not **adj(AB) = (adj A) (adj B)** (3)

(OR)

If $A = \begin{vmatrix} a & b \\ c & \frac{1+bc}{a} \end{vmatrix}$ then find A^{-1}. Also show that $aA^{-1} = (a^2 + bc + 1)I_2 - aA$, where I_2 is an identity matrix of order 2 x 2.

31. Find the distance of the point A(2, 12, 5) from the point of intersection of the line (3)
$\vec{r} = (2\hat{\imath} - 4\hat{\jmath} + 2\hat{k}) + \lambda(3\hat{\imath} + 4\hat{\jmath} + 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{\imath} - 2\hat{\jmath} + \hat{k}) = 0$.

32. Evaluate : $\int \dfrac{(x+1)}{(x+3)(x^2+4)} \, dx$ (3)

33. Solve the differential equation: $(x^2 + 1)\dfrac{dy}{dx} + 2xy = \sqrt{x^2 + 4}$ (3)

(OR)

Find the particular solution of the differential equation $x \left(\frac{dy}{dx} \right) + y - x + xy \cot x = 0$, given that $y = 0$ when $x = \frac{\pi}{2}$.

34. Find matrix A such that $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{bmatrix}$ (3)

35. Find area of the region bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.   (3)

SECTION – V

Questions 36 to 38 carry 5 marks each.

36. Three sides of a trapezium are equal, each being 6 cm long. Find the area of the trapezium when it is maximum.

(OR)

Find the intervals in which the function $f(x) = \sin 3x - \cos 3x$, $0 < x < \pi$, is
a) strictly increasing b) strictly decreasing.

37. A consulting firm rents car from three agencies such that 50% from agency L, 30% from agency M and 20% from agency N. If 90% of the cars from L, 70% of cars from M and 60% of the cars from N are in good condition,
(i) what is the probability that the firm will get a car in good condition?
(ii) If a car is in good condition, what is the probability that it has come from agency N ?

(OR)

A doctor is to visit a patient. From the past experience it is known that the probabilities of the doctor's coming by train, bus, scooter, or taxi are $\frac{1}{10}, \frac{1}{5}, \frac{3}{10}$ and $\frac{2}{5}$ respectively. The probabilities that he will be late are $\frac{1}{4}, \frac{1}{3}$ and $\frac{1}{12}$ if he comes by train, bus or scooter respectively but by taxi he will not be late. When he arrives, he is late. What is the probability that (i) he has come by bus? (ii) he has not come by bus, scooter or taxi?

38. Solve the following LPP graphically :
Maximize $Z = 50x + 60y$, subject to the constraints
$20x + 10y \le 200$, $10x + 20y \le 120$, $10x + 30y \le 150, x \ge 0, y \ge 0$.

(OR)

Solve the following LPP graphically :
Minimize $Z = 7x + 10y$, subject to the constraints
$4x + 6y \le 240$, $6x + 3y \le 240$, $x \ge 10, x \ge 0, y \ge 0$.

End of the Question Paper